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Regional Simulation Model

Model Testing and Analysis

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Model Testing

I. Numerical error analysis (WRR Lal, 2000)

II. Cell size, time step, run time analysis (ASCE HY Lal, 1998)

III. Analytical solutions used for calibration and verification (ASCE HY 127(7) Lal, 2001, 2005)

IV. Development of tools for calibration and parameter analysis (SVD, LSQ, optimization) (PEST manual, Lal, 1998)

V. Early test beds (JHE Lal, 1998c)

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I. Numerical error analysis

- Select proper time step and cell size so that the solution is useful for the intended purpose
- Provides a way to calculate model uncertainty due to numerical error
- Used in sizing model applications

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No numerical model is perfect

- Models approximate complicated systems
- Models leave out many components depending on discretization, design, and use

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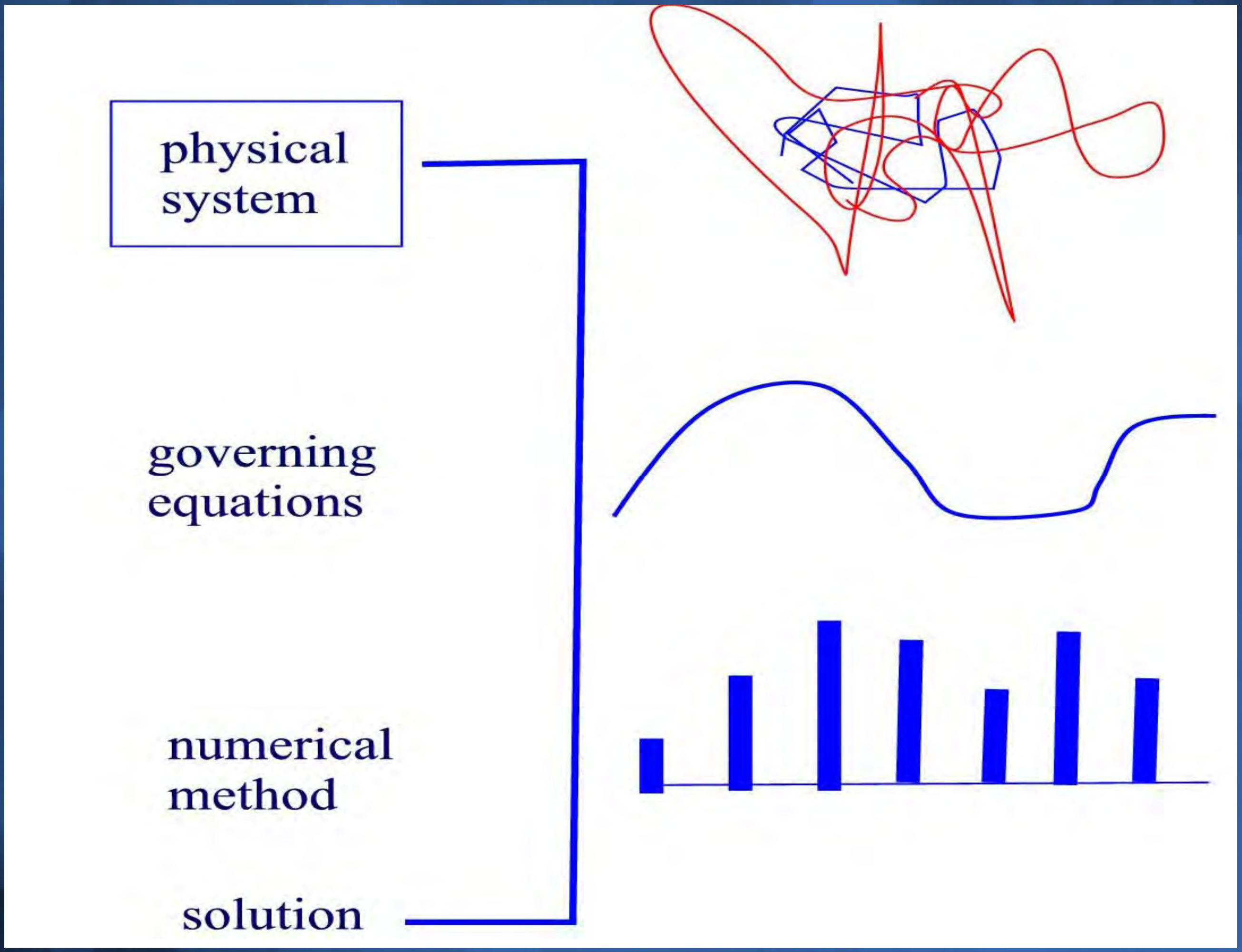
physical
system
governing
equations
numerical
method
solution

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Numerical error and stability analysis

- **Fourier decomposition and Von Neumann analysis**
  - **Models filter out Fourier components due to numerical diffusion**
  - **Models add Fourier components due to numerical dispersion**

Numerical errors are due to:

(A) Representation of continuous functions using discrete methods

(B) Insufficient space and time discretizations

(C) Computational errors due to propagation of truncation error

(Lal, WRR May 2000)

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Numerical models have performance curves

- No free lunch; for the same mesh, accurate results require small Δt and long run times
- To see more spatial details, use small Δx and pay a very high price
- Optimal discretization is based on benefit-cost analysis

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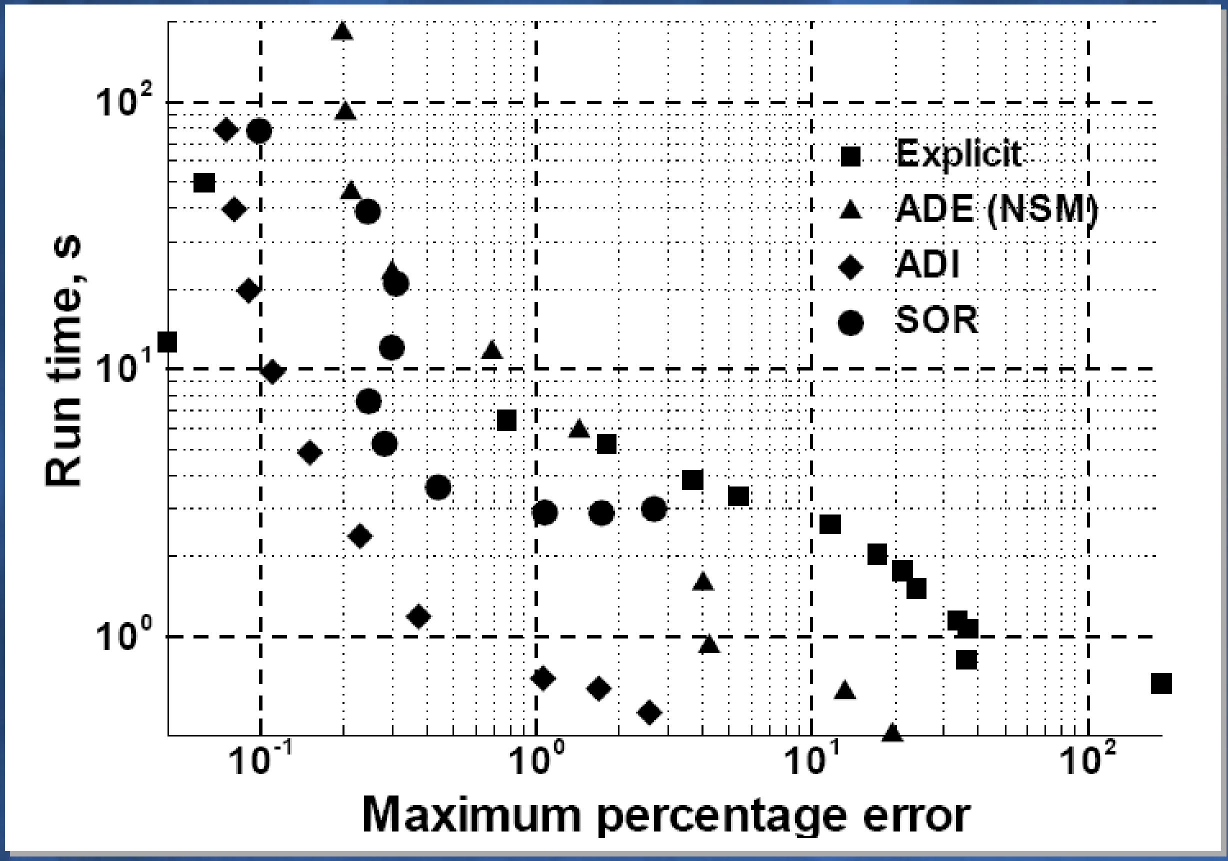
II. Model Performance



Lal, 1998, ASCE HY 124(4)

Estimate run time as (Lal, 1998 ASCE HY 124(4))

$$t_r = \frac{c_1 T K A k^4}{\psi \phi^2}$$

$$t_r = c_1 \frac{T}{\beta} \frac{(NM)^2 K}{L_x L_y} = \frac{c_1 T K A k^4}{\beta \phi^4}$$

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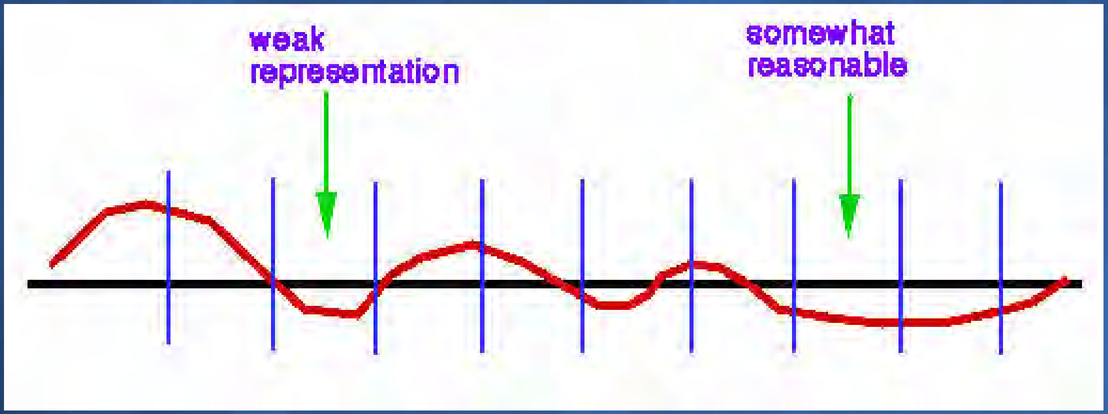
How to measure the level of discretization?



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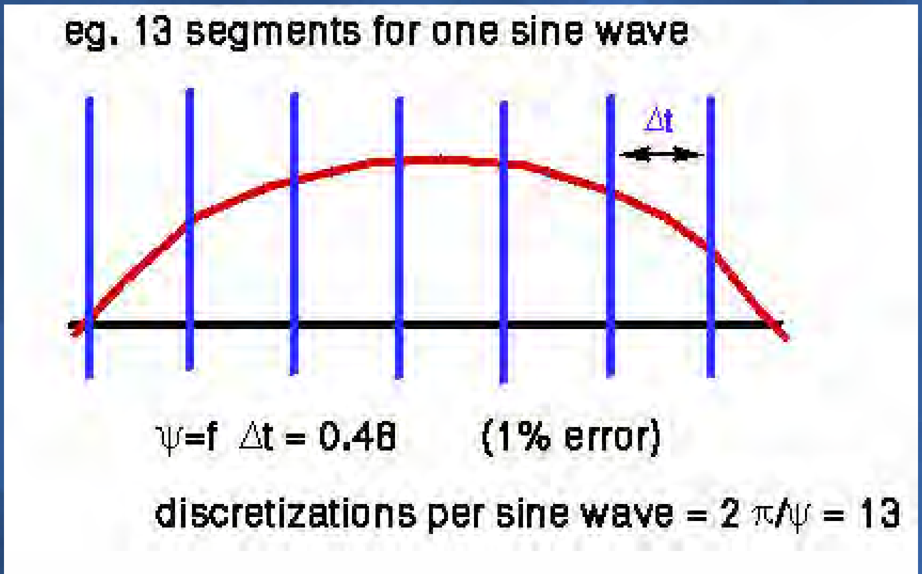
Dimensionless discretization



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(A) Errors representing continuous functions using discrete values

$$\phi = 0.5\sqrt{\epsilon_d} \quad \text{or} \quad \epsilon_d = 4.0\,\phi^2 \quad \text{for 1-D}$$

$$\phi = 0.35\sqrt{\epsilon_d} \quad \text{or} \quad \epsilon_d = 7.8\,\phi^2 \quad \text{for 2-D}$$

$\phi = k\Delta x =$ dimensionless spatial discretization.

$\psi = f\Delta t =$ dimensionless time step

$k = \frac{2\pi}{\lambda} =$ wave number

$f = \frac{2\pi}{T} =$ frequency

$\lambda =$ wave length.

$T =$ wave period.

$\epsilon_d =$ discretization error, %.

(B) Insufficient space and time discretizations

$$f = Kk^2 \qquad \text{for 1-D}$$

$$f = 2Kk^2 \qquad \text{for 2-D}$$

For overland flow in South Florida, $K = \frac{h^{\frac{5}{3}}}{n_b \sqrt{S}}$.

or $K = 250\ m^2/s$

For groundwater flow in South Florida, $K = 2\ m^2/s,\ s_c = 0.25$.

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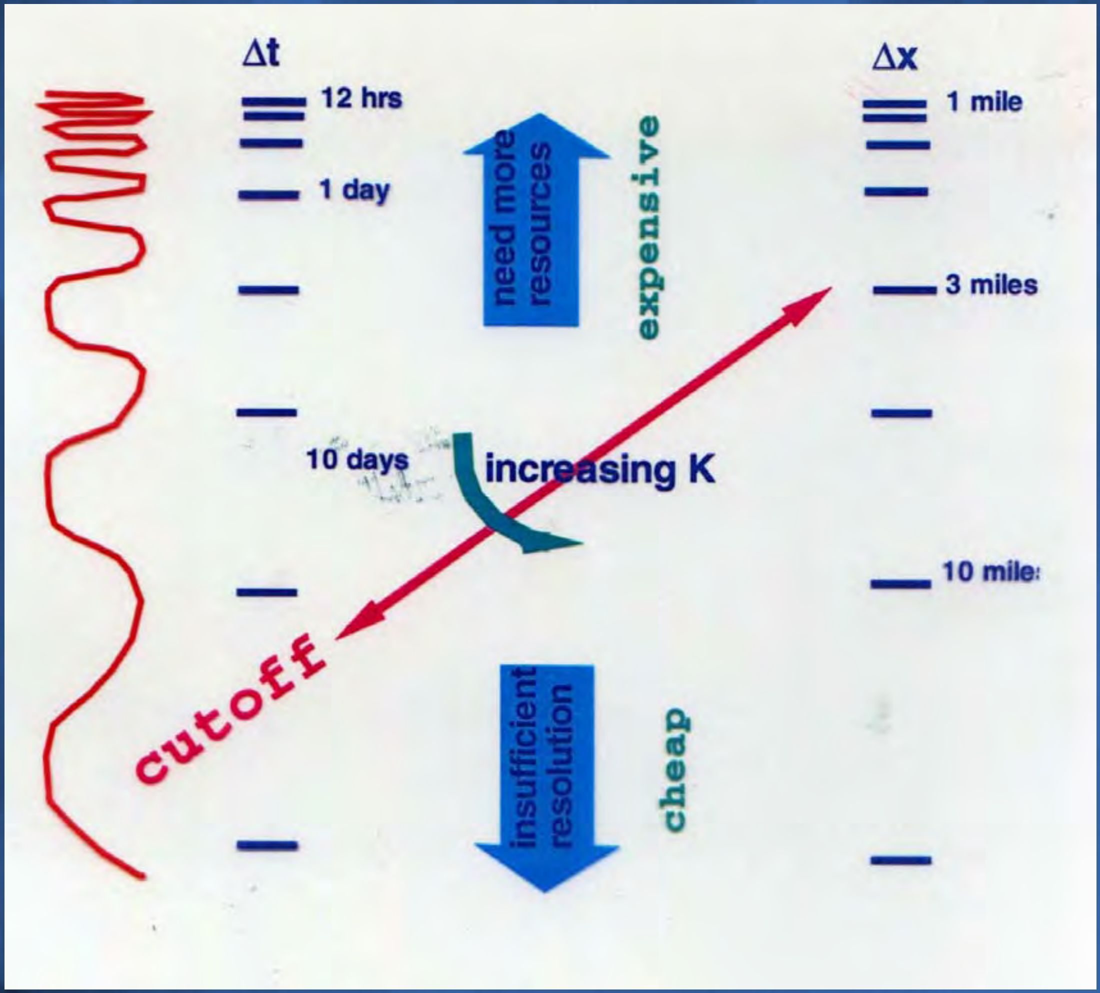
Matching of Resolutions



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(C) Computational errors

$$\epsilon_T \quad \text{(expl/impl 1-D)} \quad \approx \frac{fT\phi^2}{2}(\mp\beta - \frac{1}{6})$$

$$\epsilon_T \quad \text{(semi-impl 1-D)} \quad \approx fT\left[\frac{\phi^2}{12} - \frac{\phi^4}{12}(\beta^2 - \frac{1}{30})\right]$$

$$\epsilon_T \quad \text{(expl/impl 2-D)} \quad \approx fT\phi^2(\pm\beta - \frac{1}{12})$$

$$\epsilon_T \quad \text{(semi-impl 2-D)} \quad \approx fT\left[-\frac{\phi^2}{6} + \frac{2\phi^4}{3}(\beta^2 + \frac{1}{120})\right]$$

$\phi = k\Delta x =$ dimensionless spatial discretization.

$\beta = \frac{K\Delta t}{\Delta x^2} =$ dimensionaless time step.

$K = \frac{h^{\frac{5}{3}}}{n_b\sqrt{S}}$ for overland flow.

$\epsilon_T =$ error as a fraction of the amplitude.

Lal, 2000 WRR 36(5)

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III. Analytical solutions used for calibration and verification

- **Critical Guiding Principles:**
  - **Understanding and controlling dimensional parameter groups**
  - **Understanding the range of validity**
  - **Analytical solution is the best tool to test and verify the numerical method**
  - **Analytical solutions can be used to determine parameters**

Example where analytical solution is used to determine validity of the Diffusion Flow Assumption

TS Sqrt(g/d) >30

T = wave period

S = slope

d = depth

(Ponce, 1978)

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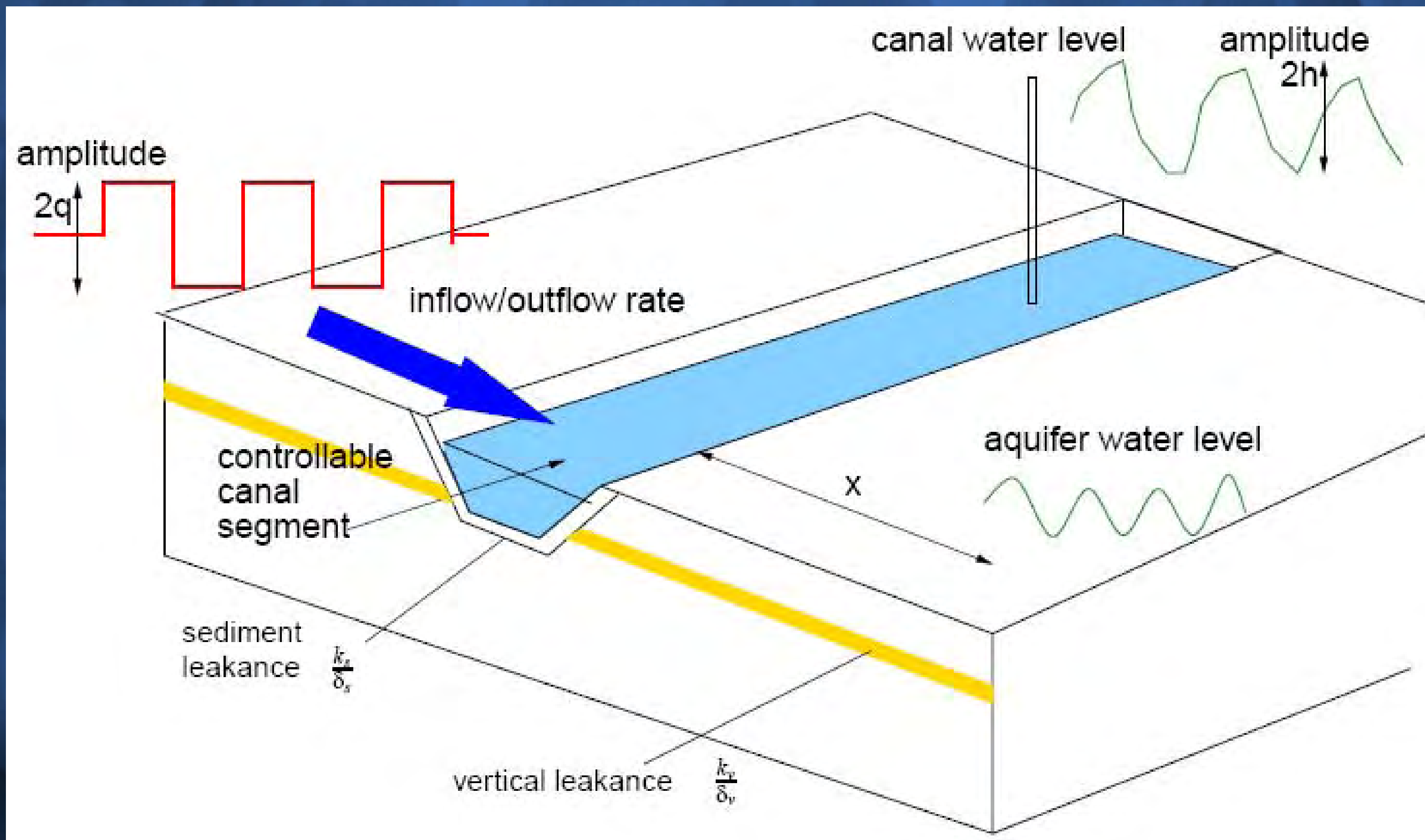
Analytical solutions for testing solutions



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Analytical solutions used to verify RSM results

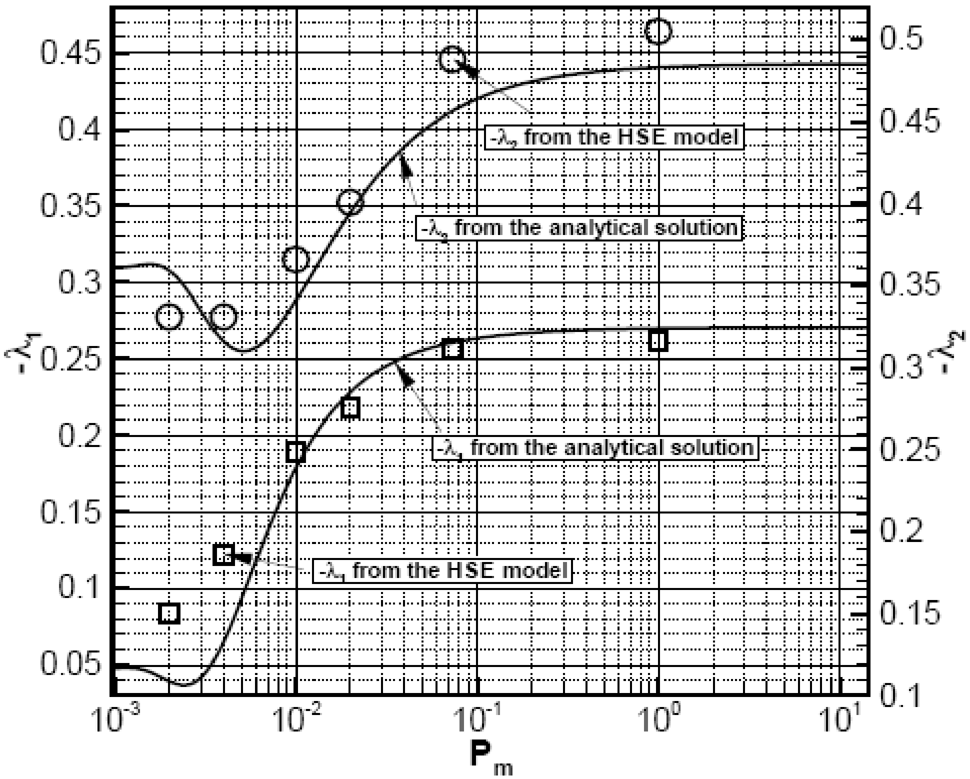

Figure 10: Comparison of the $-\hat{\lambda}_1$ versus P_m and $-\hat{\lambda}_2$ versus P_m curves obtained by using the analytical method and the HSE model. $P_d = 0.3737$, $P_r = 9.78 \times 10^{-5}$, $P_b = 2.49 \times 10^{-2}$

Lal, 2000, ASCE HY 127(7)

Analytical solutions used to understand the physical processes

Vertical leakance

Parameter describing vertical leakiness of the confined aquifer η: aquifer is confined if $\eta < 0.05$ and extremely leaky confined if $\eta > 100$	$\eta = \frac{1}{s_c f} \frac{k_v}{\delta_v}$

Sediment resistance

Dimensionless sediment conductance parameter σ: if $\sigma < 0.073$ there is full cutoff; if $\sigma > 19.5$ the sediment is fully pervious. Values described here are at a 5% detection level	$\sigma = p \frac{k_s}{\delta_s} \sqrt{\frac{2}{f s_c T}}$

Canal-aquifer storage parameter

Dimensionless stream-aquifer interaction parameter χ: with no sediments, if $\chi > 27.5$, there is cutoff and therefore no interaction. If $\chi < 0.1$, there is full interaction and the canal and the aquifer move in unison. Values described here are at a 5% detection level.	$\chi = B \sqrt{\frac{f}{T s_c}}$

Lal (Submitted to WRR)

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Analytical methods used to determine parameters

Table 4: Table of dimensionless parameters

Domain	Overall	Overall	Overall	Zone NTS1,NTS10
Method	LSQ	Cross corr	Manual	LSQ
η	0.635	1.040	0.182	3.058
k_0	2.473×10^{-4}	2.831×10^{-4}	2.636×10^{-4}	1.866×10^{-4}
σ (ampl)	2.503	2.604	2.339	3.749
σ (phase)	3.527	4.065	7.764	-
α_s (ampl)	0.629	0.617	0.650	0.653
ξ (ampl)	0.254	0.254	0.254	0.254
χ (ampl)	0.405	0.412	0.397	0.501
χ (phase)	0.688	0.844	0.452	1.332
θ	0.690	0.690	0.690	0.690

Explanations

(ampl) = Values computed using ξ or amplitude ratio of head and discharge

(phase) = Values computed using θ or the phase lag between head and discharge

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Analytical methods used to determine parameters

Table 5: Table of primitive variable computed using various methods

Domain	Overall	Overall	Overall	Zone NTS1, NTS10
Method	LSQ	Cross corr	Manual	LSQ
1. Aquifer diffusivity T/s_c, (m^2/s)	297	227	261	522
2. Ampl based diffusivity T/s_c, (m^2/s)	151	92	204	132
(assuming non-leaky)				
3. Aquifer $T\,s_c$ (ampl), (m^2/s)	0.0742	0.0716	0.0768	0.0484
4. Aquifer $T\,s_c$ (phas), (m^2/s)	0.0257	0.0170	0.0595	0.0068
5. Transmissivity T,(ampl) m^2/s	4.69	4.03	4.48	5.03
6. Transmissivity T,(phas) m^2/s	2.76	1.96	3.94	1.89
7. Storage coeff s_c	0.0158	0.0177	0.0171	0.0096
8. Coeff of leakage (sediment) k_s/δ_s (day^{-1})	13.72	14.03	13.06	16.62
9. Coeff of leakage (aquifer) k_v/δ_v (day^{-1})	0.0315	0.0581	0.0098	0.0925

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IV. Tools for calibration and parameter analysis (SVD, LSQ, Optimization)

- Sensitivity analysis
- Single Value Decomposition (SVD) is useful in determining parameter redundancies, groupings etc.; over parameterization (under determination) is a common problem
- SVD useful in determining the actual parameter dimensionality

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Tools for calibration and parameter analysis (SVD, LSQ, Optimization)

- **SVD is useful in determining parameter covariance and correlation so that parameters can be grouped**
- **SVD and LSQ (Gauss Newton) methods are useful in parameter calibration**
- **Optimization is a way to calibrate parameters**

HSE early test beds

- Axisymmetric problems
- Analytical solutions for integrated stream-aquifer problems in dimensionless terms
- Test problems from Viessman (1977) and Wang (1982) etc.
- Compare analytical and RSM estimates of error

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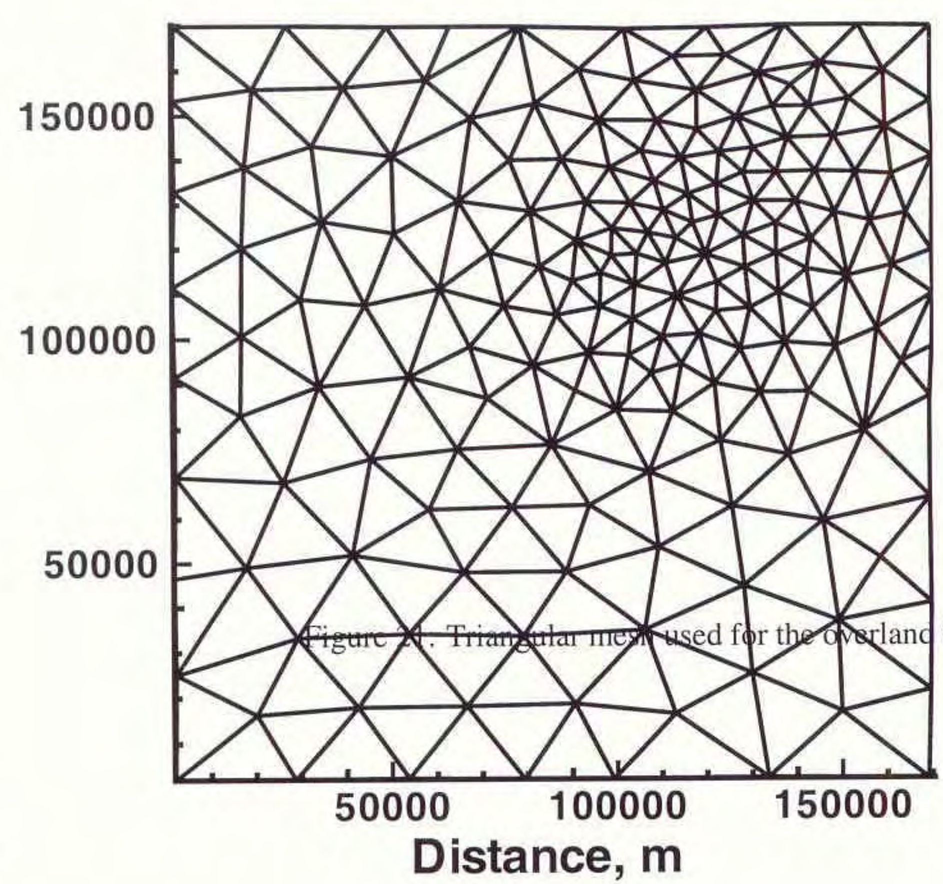

Figure 21. Triangular mesh used for the overland flow test

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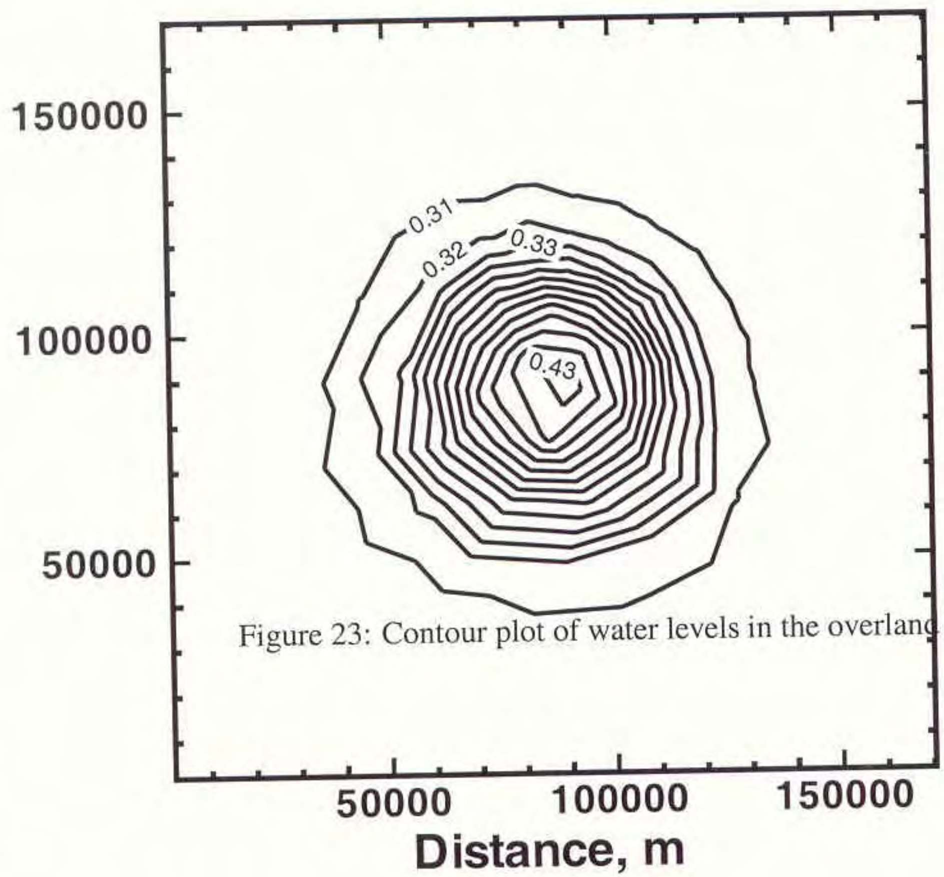

Figure 23: Contour plot of water levels in the overland flow solution

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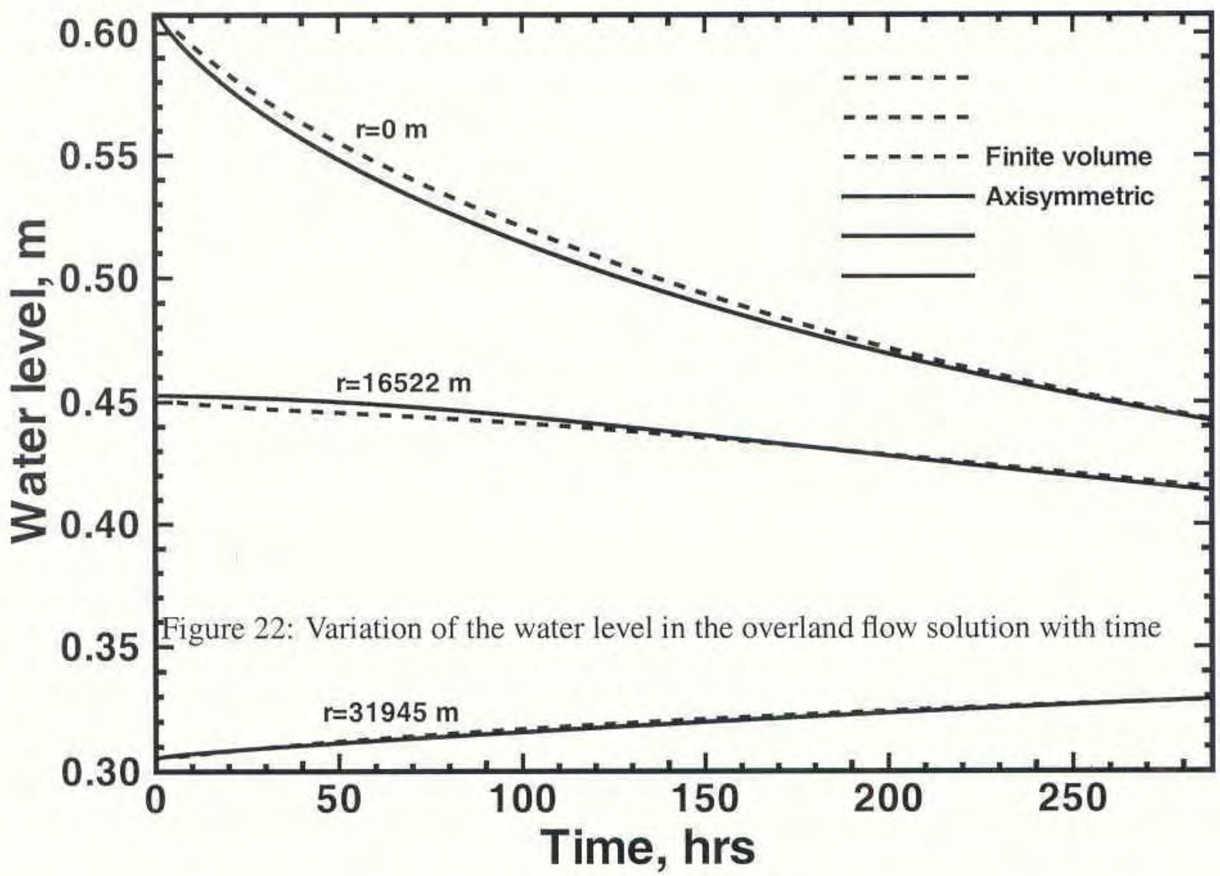

Figure 22: Variation of the water level in the overland flow solution with time

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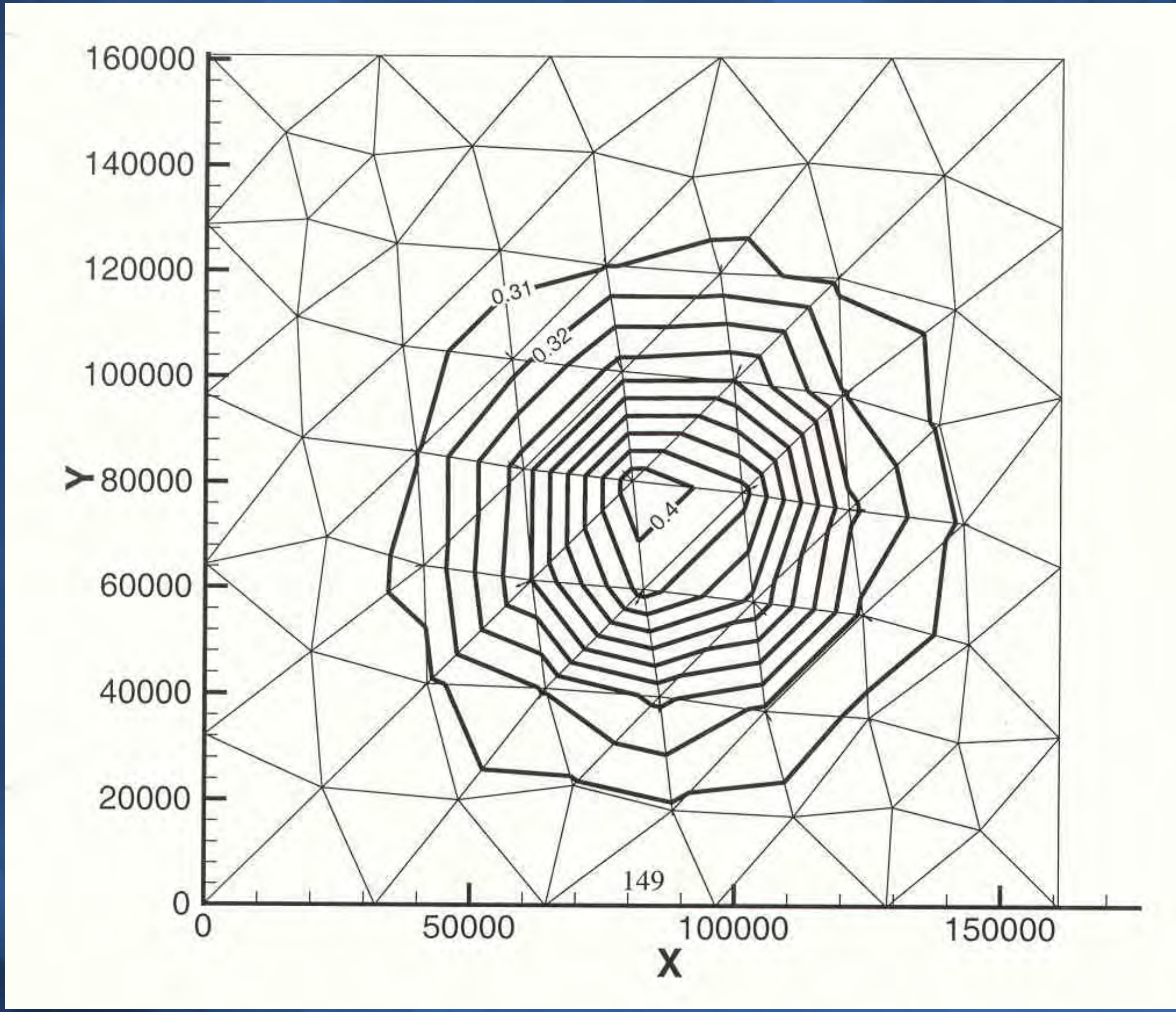
160000
140000
120000
100000
80000
60000
40000
20000
0
Y
0
50000
100000
150000
X
0.31
0.32
0.4
149

Error table

Test	No. elem.	No. nodes	CPU (s)	No. iter.	Δx (m)	Δt (s)	h_{end} (m)	π/ϕ	β	ε %
1	116	69	2.4	18	14939	51840	0.44877	2.15	0.0164	1.09
2	116	69	8.8	12	14939	10368	0.44840	2.15	0.0033	1.03
3	116	69	16.4	11	14939	5184	0.44840	2.15	0.0016	1.02
4*	238	135	10.3	1	10429	5184	0.43921			0.50
5*	238	135	15.7	1	10429	10368	0.43908			0.50
6*	238	135	27.7	1	10429	5184	0.43901		0.50	0.49
7	376	209	6.0	40	8298	207360	0.44500	3.88	0.2121	0.48
8	376	209	25.1	19	8298	20736	0.44456	3.88	0.0212	0.40
9	376	209	43.6	17	8298	10368	0.44444	3.88	0.0106	0.38
10	376	209	78.8	13	8298	5184	0.44438	3.88	0.0053	0.37
11	1536	809	60.1	104	4105	518400	0.45404	7.84	2.1660	1.96
12	1536	809	75.3	78	4105	207360	0.44494	7.84	0.8660	0.48
13	1536	809	98.3	67	4105	103680	0.44501	7.84	0.4332	0.48
14	1536	809	258.0	35	4105	20736	0.44388	7.84	0.0866	0.29
15	1536	809	436.0	27	4105	10368	0.44374	7.84	0.0433	0.27

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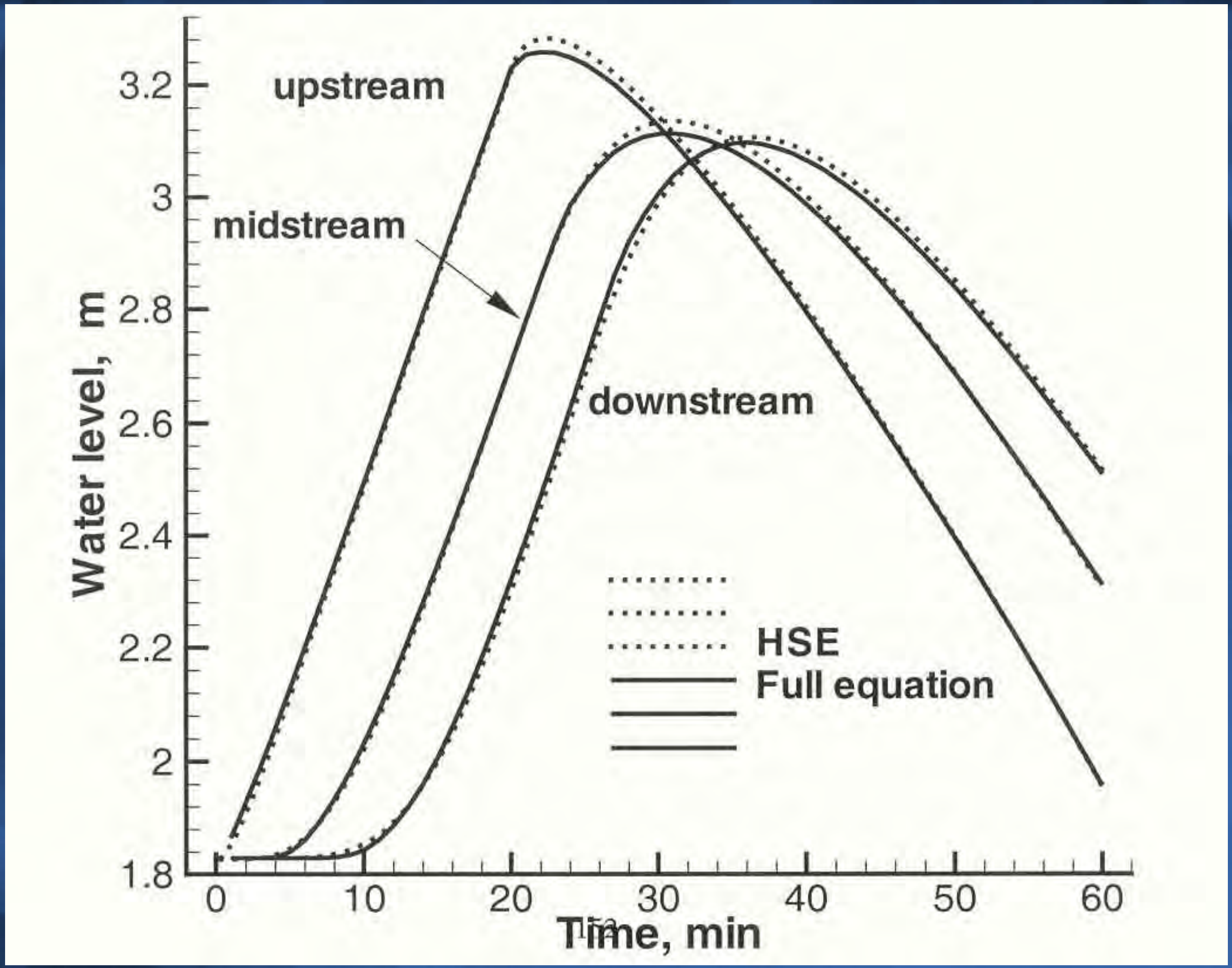
Benchmark #11



From Viessman and Knapp (1972)

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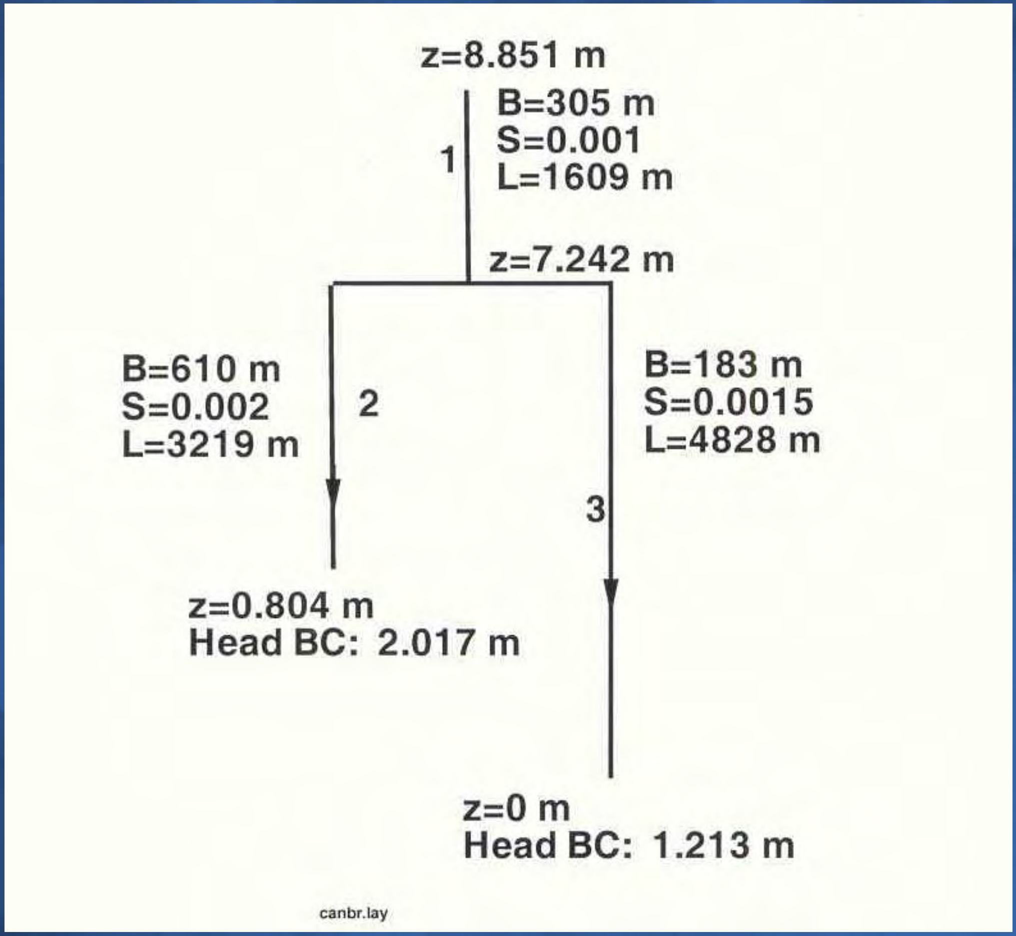
UNET Model Comparison



(USACE 1998)

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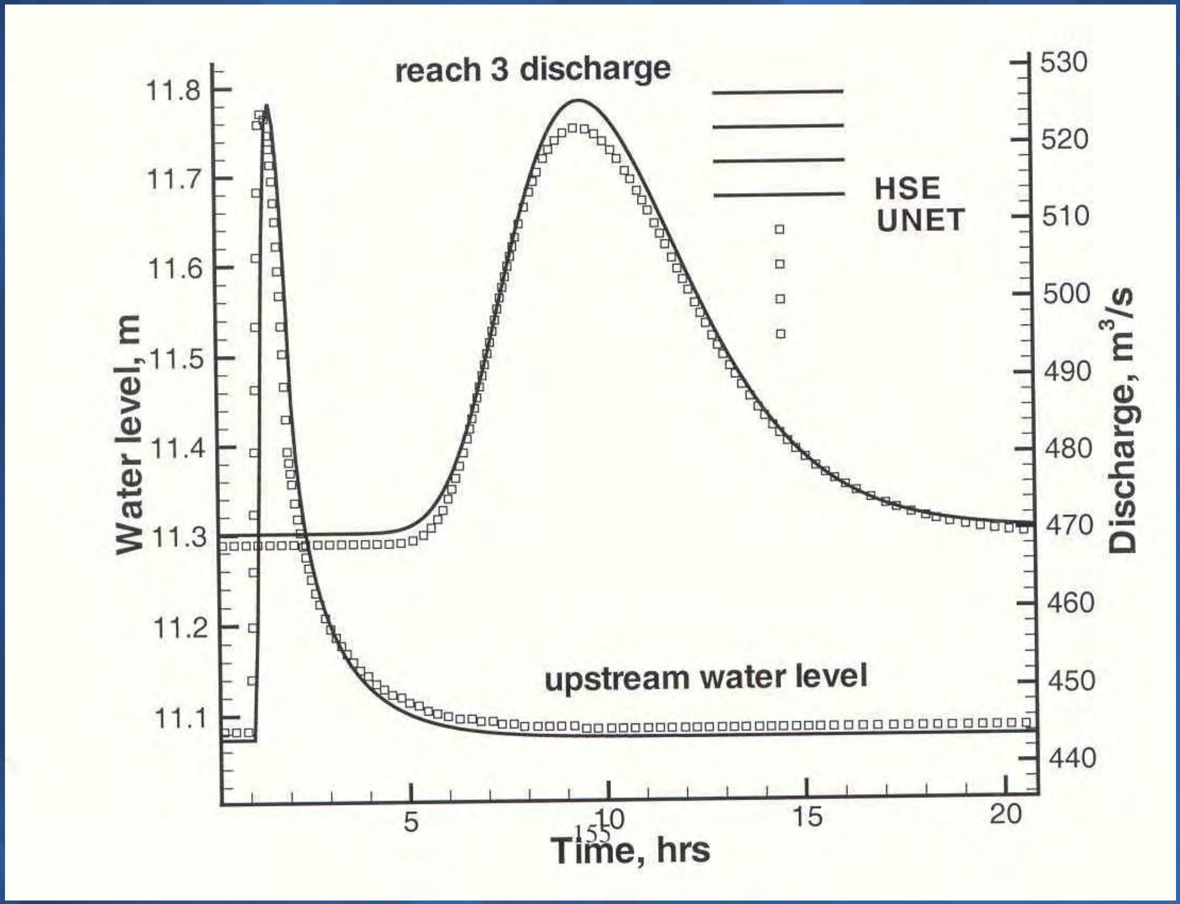
Comparison between HSE and UNET model



(USACE 1998)

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V. Early applications

Ken Tarboton will cover these early applications

- Kissimmee Basin
- Everglades National Park
- L-8 Drainage Basin
- Loxahatchee National Wildlife Reserve (WCA-1)
- Southwest Florida
- North Palm Beach County Pre-Drainage
- Southern Everglades
- South Florida (SFRSM)